Homework Solutions

S11) $|H(j\omega)| = 1 \qquad |\omega| < \omega_c$

$\qquad\qquad\quad\; 0 \qquad |\omega| > \omega_c$

a) $\measuredangle H(j\omega) = 0$

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi}\left.\frac{e^{j\omega t}}{jt}\right|_{-\omega_c}^{\omega_c} = \frac{1}{\pi t}\left[\frac{e^{j\omega_c t} - e^{-j\omega_c t}}{2j}\right]$$

$$= \frac{\sin(\omega_c t)}{\pi t}$$

b) $\measuredangle H(j\omega) = \omega T$

$$\therefore \; H(j\omega) = e^{j\omega T} \qquad |\omega| < \omega_c$$

$$\qquad\qquad\quad 0 \qquad |\omega| > \omega_c$$

$$h(t) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega T}e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(T+t)}\,d\omega$$

$$h(t) = \frac{1}{2\pi} \left. \frac{e^{j\omega(T+t)}}{j(T+t)} \right|_{-\omega_c}^{\omega_c} = \frac{1}{2\pi j(T+t)} \left[e^{j\omega_c(T+t)} - e^{-j\omega_c(T+t)} \right.$$

$$= \frac{1}{\pi(T+t)} \left[\frac{e^{j\omega_c(T+t)} - e^{-j\omega_c(T+t)}}{2j} \right]$$

$$= \frac{1}{\pi(T+t)} \sin(\omega_c(T+t))$$

same as using

$$x(t+T) \overset{\mathcal{F}}{\longleftrightarrow} e^{j\omega T} X(j\omega)$$

on the solution to part (a)

c)

$$H(j\omega) = \begin{cases} e^{j\frac{\pi}{2}} \cdot 1 = j & 0 < \omega < \omega_c \\ e^{-j\frac{\pi}{2}} \cdot 1 = -j & -\omega_c < \omega < 0 \\ 0 & |\omega| > \omega_c \end{cases}$$

$$h(t) = \frac{1}{2\pi} \int_{-\omega_c}^{0} (-j)\, e^{j\omega t} d\omega + \frac{1}{2\pi} \int_{0}^{\omega_c} (j)\, e^{j\omega t} d\omega$$

$$= \frac{-j}{2\pi} \left. \frac{e^{j\omega t}}{jt} \right|_{-\omega_c}^{0} + \frac{j}{2\pi} \left. \frac{e^{j\omega t}}{jt} \right|_{0}^{\omega_c}$$

$$= -\frac{1}{2\pi t}\left[1 - e^{-j\omega_c t}\right] + \frac{1}{2\pi t}\left[e^{j\omega_c t} - 1\right]$$

$$= \frac{1}{2\pi t}\left[e^{j\omega_c t} + e^{-j\omega_c t} - 2\right] = \frac{1}{\pi t}\left[\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2} - 1\right]$$

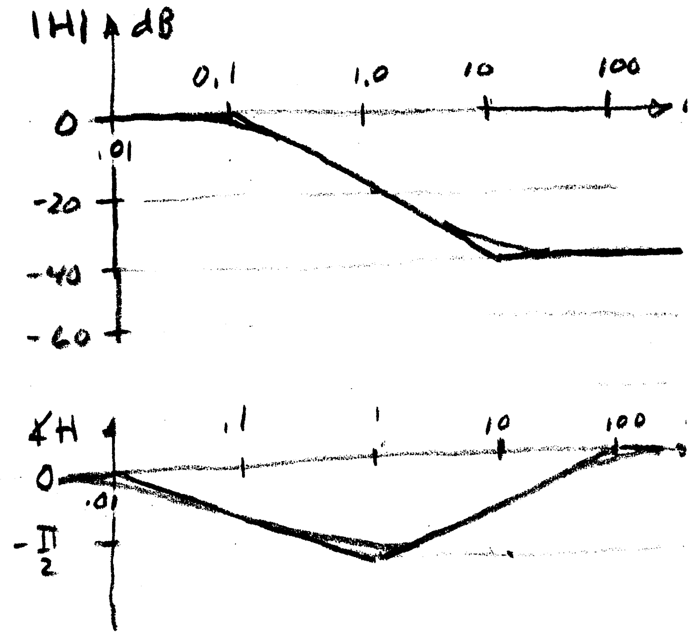
$$= \frac{1}{\pi t}\left[\cos(\omega_c t) - 1\right]$$

S12)

a) (i) $H(j\omega) = \dfrac{1 + \left(\frac{j\omega}{10}\right)}{1 + \left(\frac{j\omega}{0.1}\right)}$

magnitude reduced

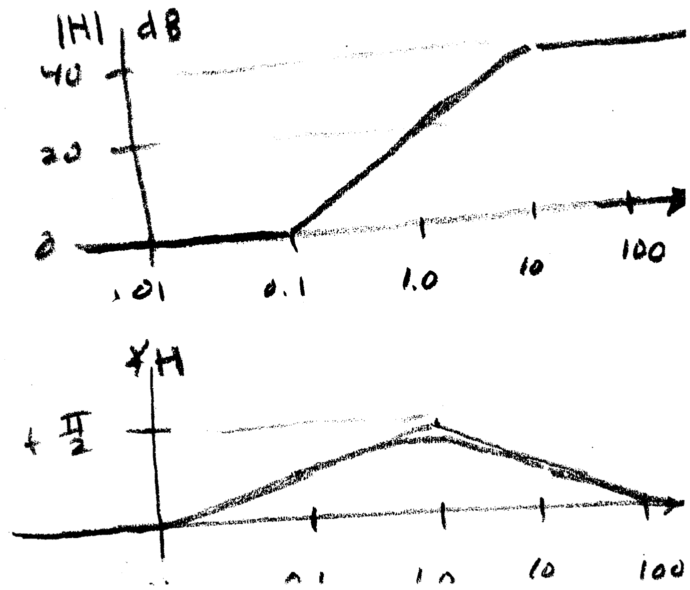
phase lag.



(ii) $H(j\omega) = \dfrac{1 + \left(\frac{j\omega}{0.1}\right)}{1 + \left(\frac{j\omega}{10}\right)}$

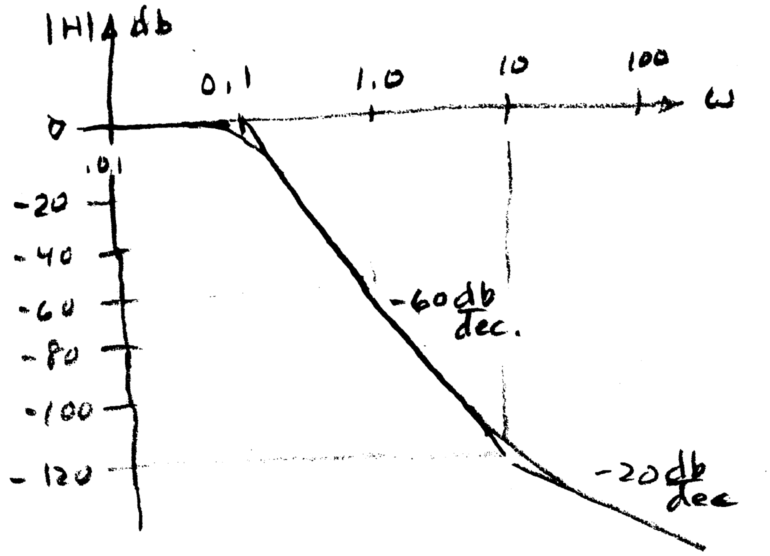
magnitude increased

phase lead

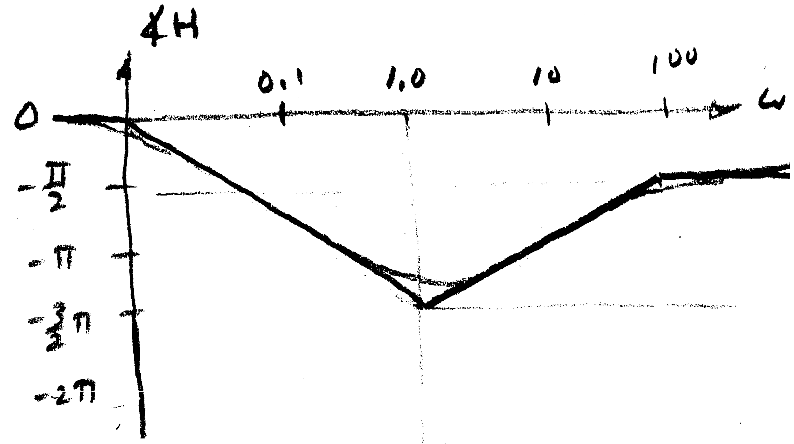


S12)(b)

(i) $H(j\omega) = \dfrac{\left(1 + \frac{j\omega}{10}\right)^2}{\left(1 + \left(\frac{j\omega}{0.1}\right)\right)^3}$

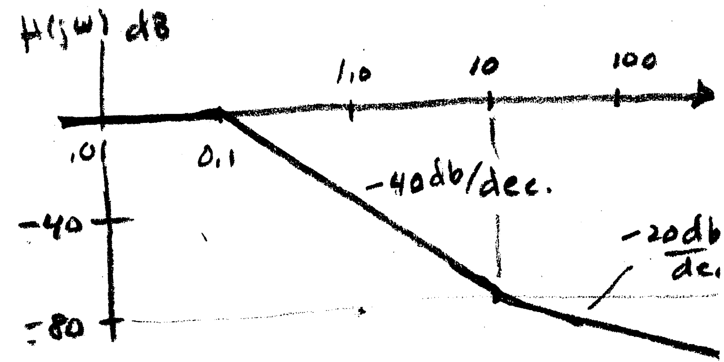


magnitude reduced



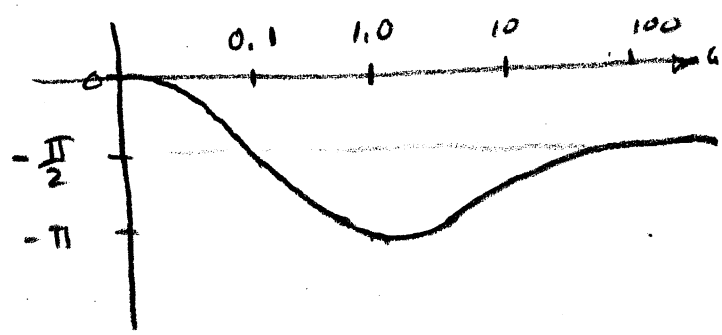
phase lag

(ii) $H(j\omega) = \dfrac{1 + \left(\frac{j\omega}{10}\right)}{\underbrace{\left(\frac{j\omega}{.1}\right)^2 + \left(\frac{j\omega}{.1}\right) + 1}_{\omega_n = .1 \quad \zeta = 0.5}}$



magnitude reduced

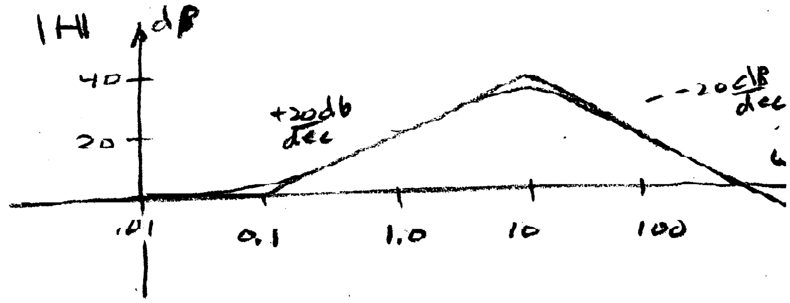
Phase lag



S12 (b)
(iii)

$\omega_n = 10 \quad \zeta = 1$

magnitude increased then reduced



Phase lead then phase lag